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TITLE- Minimum-Time Attitude Maneuvers of Spacecraft with Control Moment Gyroscopes (CMGs)

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AUTHOR(S)- J. Kranton

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ABSTRACT

The problem of executing arbitary attitude maneuvers with CMGs is investigated in response to the possibility that a Saturn V (Dry) Workshop will require such a capability. The design of a maneuver control law is formulated as a two-part problem: first, to determine the maneuver control torque, and second, to command the CMG gimbal angle rates so as to produce that torque (i.e., with no cross coupling).

The control torque is selected such that the maneuver is executed in minimum time, subject to constraints on the spin angular momentum and torque of the CMG system. A solution to this problem is presented in which the angular acceleration and angular rate about an arbitrary rotation vector are expressed as functions of the rotation angle. From this solution, a practical, suboptimal method of maneuver can be developed for implementation on future flight hardware.

The CMG gimbal angle rates required to produce the maneuver control torque are determined so as to minimize their dynamic range.

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DATE: June 24, 1969

FROM: J. Kranton

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TECHNICAL MEMORANDUM

INTRODUCTION

A maneuver capability for large, earth-orbiting spacecraft probably will be required to acquire new attitudes for particular experiments and to periodically "dump" the angular momentum accumulated by a CMG system. In general, the problem is to execute an arbitrary maneuver in as short a time as possible, subject to the constraints imposed by the CMG system.

The solution to this problem divides naturally into two parts, calculating of the required control torque and commanding the CMG gimbal angle rates to produce that torque. Practical solutions to both problems are presented in this memorandum. These solutions can form the basis for implementation on future flight hardware.

CONTROL TORQUE FOR MINIMUM-TIME MANEUVERS

Rotation Axis and Rotation Angle

Recall the so-called Euler rotation theorem; any finite rotation of a rigid body may be expressed as a rotation through some angle (rotation angle) about some fixed axis (rotation axis). In mathematical terms this theorem yields the following results.

Let A be the transformation matrix that relates a vector $(\underline{u})$ expressed in the coordinates $(X_1Y_1Z_1)$ that define the initial attitude and the coordinates $(X_2Y_2Z_2)$ that define the final attitude; that is

$$(\underline{u})_2 = A(\underline{u})_1 \tag{1}$$

Then the equation for the rotation angle Φ is

$$\Phi = \cos^{-1}\left[\frac{1}{2}\ (\mathrm{tr}\ A - 1)\right] \quad ; \quad 0 \leq \Phi \leq \pi \tag{2}$$

and the elements of the rotation vector $\underline{e}$ are given by

$$e_1 = \frac{a_{23}-a_{32}}{2s\Phi} \quad , \quad e_2 = \frac{a_{31}-a_{13}}{2s\Phi} \quad , \quad e_3 = \frac{a_{12}-a_{21}}{2s\Phi} \tag{3}$$

During the maneuver $\underline{e}$ is fixed relative to inertial coordinates and spacecraft coordinates, but the rotation angle decreases to zero. We can write

$$\phi(t) = \cos^{-1}\left[\frac{1}{2}(\mathrm{tr}\ A(t) - 1)\right] \tag{4}$$

where $\phi(0) = \Phi$. Letting T be the total maneuver time, we have $\phi(T) = 0$ and $A(T) = U$, the identity matrix.

In what follows it will be convenient to use an angle λ, where

$$\lambda(t) \equiv \Phi - \phi(t)$$

so that $\lambda(0) = 0$ and $\lambda(T) = \Lambda \equiv \Phi$.

Minimum-Time Maneuvers

Having $\underline{e}$ and Λ, we wish to execute the maneuver in minimum time, subject to the constraint that the angular momentum the CMG system can provide is not exceeded. Observe that the maneuver time is given by

$$T = \int_0^{\Lambda} \frac{d\lambda}{\dot{\lambda}}$$

Thus, T will be minimized if at all times $\dot{\lambda}$ is maximized, subject to the angular momentum constraint of the CMG system.

To determine the maximum $\dot{\lambda}$, assume for the moment that the total angular momentum of the spacecraft and CMGs remains unchanged during the maneuver; that is, the effect of external torques are neglected during the maneuver. We write therefore

$$\underline{H}(0) = \underline{H}(t) + I\,\underline{\Omega} = \underline{H}(t) + \dot{\lambda}\, I\, \underline{e} \qquad (5)$$

where $\underline{H}(0)$ and $\underline{H}(t)$ are the total CMG spin angular momenta at times 0 and t respectively, I is the inertia matrix of the spacecraft, and $\underline{\Omega}(=\dot{\lambda}\underline{e})$ is the angular velocity of the spacecraft. Note that $\underline{H}(0)$ and $\underline{e}$ in (5) have a fixed orientation relative inertial coordinates, but $\underline{H}(t)$ and I $\underline{e}$ do not.

For convenience, let us introduce the following notation:

$$\underline{H} = |\underline{H}|\underline{h} \equiv H\,\underline{h} \quad , \quad \dot{\lambda}\, I\, \underline{e} \equiv \dot{\lambda}\, |I\,\underline{e}|\underline{w} \equiv \dot{\lambda}\, W\, \underline{w}$$

where $\underline{h}$ and $\underline{w}$ are unit vectors. Using these definitions, we can rewrite (5) as

$$H(t)\underline{h}(t) = -\dot{\lambda}\, W\, \underline{w} + H(0)\underline{h}(0) \qquad (6)$$

By forming the self dot product on both sides of this equation, we get

$$H^2(t) = W^2\dot{\lambda}^2 - 2H(0)W\ c\gamma\ \dot{\lambda} + H^2(0) \tag{7}$$

where $c\gamma = \underline{h}'(0)\underline{w}$.

Now (7) is a quadratic equation in $\dot{\lambda}$ whose solution is easily shown to be

$$\dot{\lambda} = \left[H(0)c\gamma \pm \left(-H^2(0)s^2\gamma + H^2(t)\right)^{1/2}\right]\Big/W \tag{8}$$

Since we are seeking the maximum $\dot{\lambda}$, it is clear from (8) that H(t) should be made maximum and the + sign should be chosen. Furthermore, the maximum of H(t) for a system of N CMGs is Nh, where h is the spin angular momentum per CMG. Thus we have, finally,

$$\dot{\lambda} = \left[H(0)c\gamma + \left(-H^2(0)s^2\gamma + (Nh)^2\right)^{1/2}\right]\Big/W \equiv f(\lambda) \tag{9}$$

It remains only to determine $c\gamma$ as a function of λ. This can be done easily with the aid of Fig. 1, from which we get

$$\underline{w} = \begin{pmatrix} s\alpha c\lambda \\ s\alpha s\lambda \\ c\alpha \end{pmatrix} \quad \text{and} \quad \underline{h}(0) = \begin{pmatrix} s\beta c\theta \\ s\beta s\theta \\ c\beta \end{pmatrix} \tag{10}$$

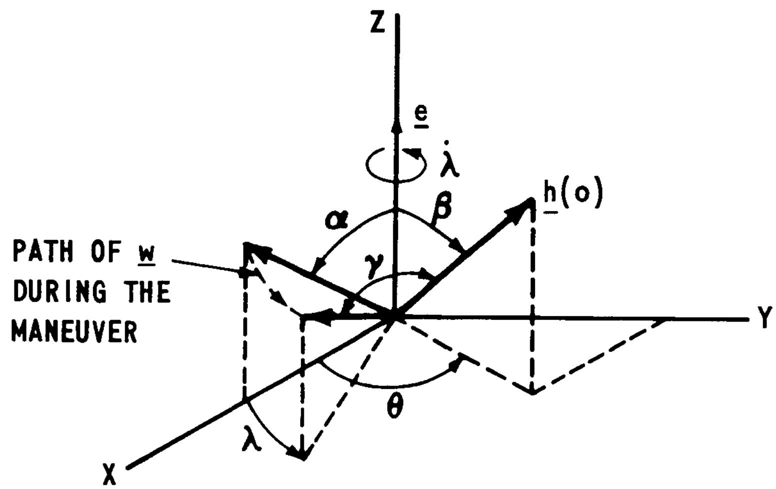

FIGURE 1 - RELATIONSHIP BETWEEN $\underline{e}$, $\underline{h}(o)$, AND $\underline{w}$ RELATIVE TO INERTIAL COORDINATES

and hence

$$c\gamma = \underline{h}'(0)\underline{w} = c\alpha c\beta + s\alpha s\beta c(\lambda - \theta) \tag{11}$$

where α, β, and θ are constant angles determined at the start of the maneuver from

$$\left.\begin{aligned}
&c\alpha = \underline{w}'\underline{e} \quad , \quad c\beta = \underline{h}'(0)\underline{e} \quad , \quad \left(0 \leq \alpha < \tfrac{\pi}{2} \ , \ 0 \leq \beta \leq \pi\right) \ ; \\
&\text{and} \\
&\theta = \theta^{+} \mathrm{sgn}\left[\underline{e}'(\underline{\tilde{w}}\ \underline{h}(0))\right]\Big|_{\lambda=0} \ ^{1} \\
&\text{where} \\
&\theta^{+} = \cos^{-1}\left[(\underline{h}'(0)\underline{w} - c\alpha c\beta)\big/ s\alpha s\beta\right]\Big|_{\lambda=0} \ , \quad \left(0 \leq \theta^{+} \leq \pi\right)
\end{aligned}\right\} \tag{12}$$

---

[1] The symbol $\sim$ over a vector indicates the cross product operation.

Certain properties of $f(\lambda)$ are evident. The extreme values of $f(\lambda)$ coincide with the extreme values of $c\gamma$, which are $c(\alpha-\beta)$ and $c(\alpha+\beta)$. These extremes occur at $\lambda=\theta$ and $\lambda=\theta+\pi$; that is, when $\underline{w}$ lies in the plane defined by $\underline{e}$ and $\underline{h}(0)$.

If α or β is zero, $s\alpha s\beta$ equals zero, and $f(\lambda)$ is constant for all λ.[1] α is zero if the maneuver is about a principal axis of the spacecraft, for then $\underline{e}$ and $\underline{w}$ are colinear. β is zero if $\underline{e}$ and $\underline{h}(0)$ are colinear. Also, if $\overline{H}(0) = 0$, $f(\lambda)$ is constant for all λ and is equal to Nh/W. Finally, note that $I_{min} \leq W \leq (I_1^2 + I_2^2 + I_3^2)^{1/2}$, where I_{min} is the minimum principal axis moment of inertia.

We have shown above that the minimum-time maneuver is achieved by employing $\dot{\lambda} = f(\lambda)$ as given by (9). But, to achieve this $\dot{\lambda}$ the angular momentum vector of the CMG system must be at its full magnitude. Thus, the tip of the CMG angular momentum vector would traverse a path on the surface of sphere of radius Nh.

Such a maneuver is impossible to realize, for it would require that $\dot{\lambda}$ jump between zero and the values given by $f(\lambda)$ at the beginning and at the end of the maneuver. That is, $\ddot{\lambda}$ would be infinite at those times, and this no CMG system could achieve. To obtain a practical scheme, a bound must be placed on $\ddot{\lambda}$; that is,

$$|\ddot{\lambda}| \leq L \tag{13}$$

With such a limit, the trajectory of a minimum-time maneuver in the phase plane has a shape such as shown in Fig. 2.

Bounds on the maneuver time may be obtained readily. To do so, let us define $\dot{\lambda}_m$ and $\dot{\lambda}_n$ as the maximum and minimum values of $\dot{\lambda}$ as determined from $f(\lambda)$ for λ in the interval $[0,\Lambda]$.

Then

$$\left(\frac{\Lambda}{\dot{\lambda}_m} + \frac{\dot{\lambda}_m}{L}\right) < T < \left(\frac{\Lambda}{\dot{\lambda}_n} + \frac{\dot{\lambda}_n}{L}\right) \tag{14}$$

---

[1] This is also true if β is π. But starting a maneuver with $\beta = \pi$ is undesirable.

where the upper bound is obtained by following the trajectory oabo' on Fig. 2, and the lower bound by following the trajectory ocdo'. It would be advantageous to arrange matters such that $\theta = \Lambda/2$, for then $\dot{\lambda}_m$ would occur at $\lambda = \Lambda/2$, thereby lowering the maneuver time.

Trajectories such as ofgo' in Fig. 2 can be attained provided L is sufficiently large for the parabolic arcs to reach the curve defined by $\dot{\lambda} = f(\lambda)$. Maneuver trajectories like those shown in Fig. 3 would result for values of L that were not large enough. Also, in order to traverse the trajectory $\dot{\lambda} = f(\lambda)$ it is necessary that

$$|\ddot{\lambda}| = \left|\frac{df(\lambda)}{d\lambda} f(\lambda)\right| \leq L \tag{15}$$

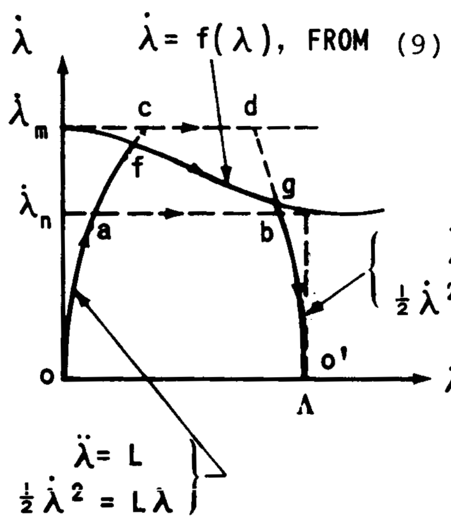

FIGURE 2

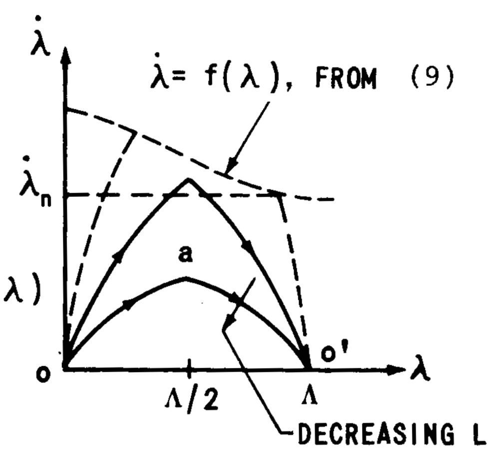

FIGURE 3
(DECREASING VALUES OF L)

PHASE PLANE TRAJECTORIES OF MINIMUM-TIME MANEUVERS

The control torque $\underline{T}_c$[1] that will produce rotation only about $\underline{e}$ is determined by noting that for the maneuver $\underline{\Omega} = \dot{\lambda}\,\underline{e}$ and $\dot{\underline{\Omega}} = \ddot{\lambda}\,\underline{e}$. We get

$$\underline{T}_c = -(\ddot{\lambda}\, I\, \underline{e} + \dot{\lambda}^2 \tilde{\underline{e}}\, I\, \underline{e}) \tag{16}$$

$$= -W(\ddot{\lambda}\, \underline{w} + \dot{\lambda}^2 \tilde{\underline{e}}\, \underline{w})$$

Effect of External Torques

In the preceding development the effect of external torques was neglected during the maneuver. The effect, as shown below, is to require that $(Nh)^2$ in (9) be replaced by

$$\left(Nh - \max_{t(\leq T)} \left| \int_0^t \underline{T}_{ext} d\tau \right| \right)^2$$

where the $\max\limits_{t(\leq T)}$ connotes the worst case effect of the external torques for $t \leq T$ (T = maneuver time).

To establish this result write

$$\underline{T}_{ext} = \frac{d}{d\tau}(I\underline{\Omega} + \underline{H})$$

which when integrated yields

$$H(t)\underline{h}(t) - \int_0^t \underline{T}_{ext} d\tau = -I\underline{\Omega} + \underline{H}(0) \tag{17}$$

$$= -\dot{\lambda} W \underline{w} + H(0)\underline{h}(0)$$

---

[1] The control torque is defined here as $\dot{\underline{H}}(t)$, the rate of change spin angular momentum of the CMG system.

This equation is identical to (6) except for the term on the left involving the integral of the external torques. Thus the left side of (7) should be replaced by the square of the magnitude of the left side of (17). Then we can write

$$\begin{aligned} \left|H(t)\underline{h}(t) - \int_0^t \underline{T}_{ext}\, d\tau\right|^2 &= \left|Nh\underline{h}(t) - \int_0^t \underline{T}_{ext} d\tau\right|^2 \\ &\geq \left(Nh - \left|\int_0^t \underline{T}_{ext} d\tau\right|\right)^2 \\ &\geq \left(Nh - \max_{t(\leq T)}\left|\int_0^t \underline{T}_{ext} d\tau\right|\right)^2 \end{aligned} \qquad (18)$$

where the $\max_{t(\leq T)}$ connotes the worst case experience during the various maneuvers required of the spacecraft.

Finally, the external torques can be taken into account in the calculation of the control torques; (16) becomes

$$\underline{T}_c = \underline{T}_{ext} - (\ddot{\lambda} I \underline{e} + \dot{\lambda}^2 \underline{\tilde{e}}\ I\ \underline{e}) \qquad (19)$$

Procedure for Executing Minimum-Time Maneuvers

A practical scheme for executing arbitrary maneuvers in near minimum time can be implemented using the concepts developed in the preceding sections. The scheme is based on using λ, $\dot{\lambda}$ trajectories of the type oabo' in Fig. 2. A step by step procedure is as follows:

1. Determine the transformation matrix A that defines the maneuver.

2. Obtain the maneuver angle $\Phi(=\Lambda)$ from (2) and $\underline{e}$ from (3).

3. Determine W, $\underline{w}$, H(0), and $\underline{h}$(0) using the definitions in (6).

4. Obtain α, β, and θ from (12).

5. Find the value of λ in the interval $[0,\Lambda]$ for which $(\lambda-\theta)$ is maximum. This value of $(\lambda-\theta)$ will give the minimum value of $c\gamma$ from (11).

6. With the minimum $c\gamma$, determine $\dot{\lambda}_n$ from (9), replacing $(Nh)^2$ by the term given in the preceding section.

7. If $\dot{\lambda}_n > +(\Lambda L)^{1/2}$, L is not sufficiently large to achieve $\dot{\lambda}_n$ and the $\lambda,\dot{\lambda}$ trajectory will be of the type oao' in Fig. 3. For such a trajectory,

$$(\dot{\lambda})_{max} = +(\Lambda L)^{1/2} \quad \text{and} \quad T = 2\left(\frac{\Lambda}{L}\right)^{1/2} \quad ;$$

$$\text{and for} \quad 0 \leq \lambda \leq \frac{\Lambda}{2} \;, \quad \ddot{\lambda} = L \;, \quad \dot{\lambda} = (2L\lambda)^{1/2} \;;$$

$$\text{while for} \quad \frac{\Lambda}{2} < \lambda \leq \Lambda \;, \quad \ddot{\lambda} = -L \;, \quad \dot{\lambda} = (2L(\Lambda-\lambda))^{1/2}.$$

8. If $\dot{\lambda}_n > +(\Lambda L)^{1/2}$, L is sufficiently large to achieve $\dot{\lambda}_n$ and the $\lambda,\dot{\lambda}$ trajectory will be of the type oabo' in Fig. 2. For such a trajectory,

$$T = \left(\frac{\Lambda}{\dot{\lambda}_n} + \frac{\dot{\lambda}_n}{L}\right) < 2\left(\frac{\Lambda}{L}\right)^{1/2}$$

Here $\ddot{\lambda}$ and $\dot{\lambda}$ are determined as follows:

$$0 \leq \lambda \leq \frac{\dot{\lambda}_n^2}{2L} \quad , \quad \ddot{\lambda} = L \quad , \quad \dot{\lambda} = +(2L\lambda)^{1/2} ;$$

$$\frac{\dot{\lambda}_n^2}{2L} < \lambda \leq \Lambda - \frac{\dot{\lambda}_n^2}{2L} \quad , \quad \ddot{\lambda} = 0 \quad , \quad \dot{\lambda} = \dot{\lambda}_n \quad ;$$

$$\Lambda - \frac{\dot{\lambda}_n^2}{2L} < \lambda \leq \Lambda \quad , \quad \ddot{\lambda} = -L \quad , \quad \dot{\lambda} = +(2L(\Lambda-\lambda))^{1/2}$$

(In steps 7 and 8, λ is calculated from $\lambda = \Phi - \phi$, where ϕ is given by (4).)

9. Calculate the maneuver control torque from (19).

Discussion

During an actual mission, $\underline{H}(0)$ can be predicted into the future because the external torques acting on a spacecraft are reasonably well known. By using the maneuver control technique developed in the previous section, the maneuver time T for a particular maneuver can also be forecast. Hence, when execution of the maneuver has been decided upon, the best time in the future to execute it can be determined. Or, if as in the LM-ATM mission, only preplanned maneuvers are to be executed every orbit, the variation in $\underline{H}$ prior to the maneuvers can be preselected (by proper initialization of CMG spin axes) such that at the beginning of a maneuver $\underline{H}(0)$ is best situated for the maneuver, that is, $\theta = \Lambda/2$.

The value of L used should be determined, in simulation studies, such that constraints on gimbal angle rates are not violated for the various maneuvers to be executed.

COMMANDED GIMBAL ANGLE RATES TO PRODUCE AN ARBITRARY CONTROL TORQUE

The control torque $\underline{T}_c$ of a CMG system is $\dot{\underline{H}}$,[1] the total rate of change of the system spin angular momentum. For any physical arrangement of CMGs on a spacecraft, the general

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[1] The time derivative here is relative to an observer in inertial coordinates.

expression for $\underline{T}_c$ is

$$\underline{T}_c = \dot{\underline{H}} = h[G\dot{\underline{\alpha}} + F\dot{\underline{\beta}} + \tilde{\underline{\Omega}}\ \underline{h}^T] \tag{20}$$

where

h = magnitude of spin angular momentum for each CMG

G,F = $3 \times N$ [1] matrices of gimbal angles

$\dot{\underline{\alpha}}, \dot{\underline{\beta}}$ = $N \times 1$ column vectors of outer and inner gimbal angle rates respectively

$\tilde{\underline{\Omega}}\ \underline{h}^T$ = cross product of the spacecraft angular velocity and the per unit total spin angular momentum of the system (i.e., $\underline{H} = h\ \underline{h}^T$).

Any solution for $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ that satisfies (20) is all that is required to produce $\underline{T}_c$ exactly, with no cross-coupling (i.e., no torques along directions other than $\underline{T}_c$). A direct solution for $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ cannot be obtained from (20) because there are 2N unknowns in 3 equations. The basic problem then is to introduce a constraint between $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ to reduce the number of unknowns in (20) from 2N to 3 so that a solution is possible. A straightforward approach to this problem is to introduce an objective function in $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ that we wish to minimize and treat (20) as a set of contraint equations. The objective function we will choose is

$$J = \frac{1}{2}\ (\dot{\underline{\alpha}}'\dot{\underline{\alpha}} + g\ \dot{\underline{\beta}}'\dot{\underline{\beta}}) \tag{21}$$

which reflects our interest in minimizing the dynamic range of the gimbal angle rates. The scalar g is positive and is to be chosen by the system designer.

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[1] N two-degree-of-freedom CMGs are assumed.

This minimization problem can be solved by the method of Lagrange multipliers. Let $\underline{\lambda}$ be a 3×1 vector of multipliers; then we can replace the problem of minimizing (21) subject to the constraint (20) with the probelm of minimizing

$$J^* = \frac{1}{2}\left(\underline{\dot{\alpha}}'\underline{\dot{\alpha}} + g\ \underline{\dot{\beta}}'\underline{\dot{\beta}}\right) + \underline{\lambda}'\left(G\ \underline{\dot{\alpha}} + F\ \underline{\dot{\beta}} + \underline{\tilde{\Omega}}\ \underline{h}^T - \frac{1}{h}\ \underline{T}_c\right) \tag{22}$$

Since (21) is a positive definite function, the necessary and sufficient condition for a minimum of (22) is that the partial derivatives with respect to the elements of $\underline{\dot{\alpha}}$ and $\underline{\dot{\beta}}$ all be zero. That is,

$$\underline{\dot{\alpha}} = -G'\ \underline{\lambda} \quad \text{and} \quad \underline{\dot{\beta}} = -g\ F'\ \underline{\lambda} \tag{23}$$

By substituting these equations into (20), a solution for $\underline{\lambda}$ is obtained as

$$\underline{\lambda} = -(G\ G' + gF\ F')^{-1}\left(\frac{1}{h}\ \underline{T}_c - \underline{\tilde{\Omega}}\ \underline{h}^T\right) \tag{24}$$

Hence,

$$\underline{\dot{\alpha}} = G'(G\ G' + gF\ F')^{-1}\left(\frac{1}{h}\ \underline{T}_c - \underline{\tilde{\Omega}}\ \underline{h}^T\right) \tag{25}$$

$$\underline{\dot{\beta}} = gF'(G\ G' + gF\ F')^{-1}\left(\frac{1}{h}\ \underline{T}_c - \underline{\tilde{\Omega}}\ \underline{h}^T\right) \tag{26}$$

The scalar g is to be chosen by the system designer to equalize, if possible, the dynamic range of the inner and outer gimbal rates as observed in simulations of system response to worst case control situations.

The above solution for $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ requires that the matrix (G G' + F F') be of rank 3. Physically this means that for an arbitrary vector $(\frac{1}{h}\, \underline{T}_c - \underline{\tilde{\Omega}}\, \underline{h}^T)$ there is some $\dot{\underline{\alpha}}$, $\dot{\underline{\beta}}$ that solves (20). But this requirement would exist for any procedure that attempts to solve (20). Hence, the solution has introduced no special restrictions on system performance.

CONCLUSION

A minimum-time maneuver is executed by rotating the spacecraft about the fixed axis defined by Euler's rotation theorem. The attainable rotation rate about this axis is constrained by the spin angular momentum of the CMG system and by the integral of the external torques that act during the maneuver. The rotational acceleration is constrained by the torque the CMGs can provide. Analytically, these constraints are introduced readily (see (18) and (13)). Practically, simulation studies are required to determine both the maximum value of the integral of the external torques and the bound on the angular acceleration for the class of maneuvers to be conducted with a given CMG system. Once these constraints are determined, the procedure for executing an arbitrary maneuver in near minimum time follows directly.

Implementing the calculations for the maneuver control torque and for the gimbal angle rates needed to produce that torque would require an on-board digital computer.

1022-JK-ep

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